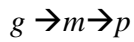


## Hw1 solution

3. In class, we introduced a simple model of gene expression:

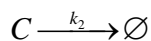
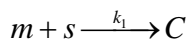
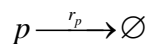
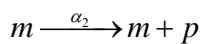
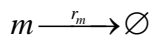
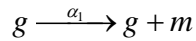


Here  $g$  is the gene (DNA),  $m$  is messenger RNA, and  $p$  is the protein and reactions occur at rates  $\alpha_1$  and  $\alpha_2$  for the first and second reaction, respectively. (The complete model also contains terms for degradation of protein and mRNA.)

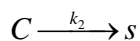
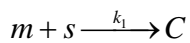
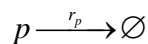
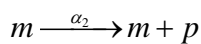
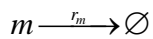
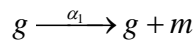
(a) Now, assume that there is a small RNA ( $s$ ) which can hybridize to the messenger RNA ( $m$ ), and degrade it.

(1) Write a reaction equation for this system assuming that the small RNA  $s$  (i) is itself degraded or (ii) is not degraded in the process.

(i)



(ii)



(2) Derive the differential equations for both systems

(i)

$$\dot{[g]} = 0$$

$$\dot{[m]} = \alpha_1 [g] - r_m [m] - k_1 [m][s]$$

$$\dot{[p]} = \alpha_2 [m] - r_p [p]$$

$$\dot{[s]} = -k_1 [m][s]$$

$$\dot{[C]} = k_1 [m][s] - k_2 [C]$$

(ii)

$$\dot{[g]} = 0$$

$$\dot{[m]} = \alpha_1 [g] - r_m [m] - k_1 [m][s]$$

$$\dot{[p]} = \alpha_2 [m] - r_p [p]$$

$$\dot{[s]} = -k_1 [m][s] + k_2 [C]$$

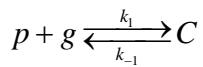
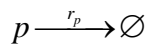
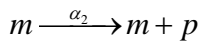
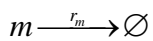
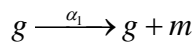
$$\dot{[C]} = k_1 [m][s] - k_2 [C]$$

(b) A transcriptional repressor is a protein that can inhibit transcription by binding to a gene. Assume that the protein p in the model above is a repressor that can bind to gene g.

(1) What chemical reactions can you use to model such a feedback system?

(2) Write down the differential equations for this system

(1)



(2)

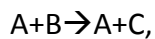
$$\dot{[g]} = -k_1 [p][g] + k_{-1} [C]$$

$$\dot{[m]} = \alpha_1 [g] - r_m [m]$$

$$\dot{[p]} = \alpha_2 [m] - r_p [p] - k_1 [p][g] + k_{-1} [C]$$

4. Solving differential equations.

(a) A catalytic reaction is modeled as follows:



$$A(t=0) = A_0, B(t=0) = B_0$$

Write the differential equations for this system, solve them, and plot B as a function of time

$$\frac{d[A]}{dt} = -k[A][B] + k[A][B] = 0$$

$$\frac{d[B]}{dt} = -k[A][B]$$

$$\frac{d[C]}{dt} = k[A][B]$$

Since  $d[A]/dt = 0$ , then  $[A] = A_0$

$$\frac{d[B]}{dt} = -kA_0[B]$$

$$\rightarrow \frac{d[B]}{[B]} = -kA_0 dt$$

$$\rightarrow \int \frac{d[B]}{[B]} = -\int kA_0 dt$$

$$\rightarrow \ln[B] = -kA_0 t + C$$

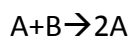
$$\rightarrow [B] = e^{-kA_0 t + C}$$

$$t = 0, [B] = B_0$$

$$\rightarrow B_0 = e^C$$

$$\rightarrow [B] = B_0 e^{-kA_0 t}$$

(b) An auto-catalyst system is modeled as follows:



$$A(t=0)=A_0, B(t=0)=B_0$$

Write the differential equations for this system, solve them, and plot B as a function of time.

$$\frac{d[A]}{dt} = -k[A][B] + 2k[A][B] = k[A][B]$$

$$\frac{d[B]}{dt} = -k[A][B]$$

By conservation law,  $[A] + [B] = A_0 + B_0$

$$\rightarrow [A] = A_0 + B_0 - [B]$$

$$\frac{d[B]}{dt} = -k(A_0 + B_0 - [B])[B]$$

$$\rightarrow \frac{d[B]}{(A_0 + B_0 - [B])[B]} = -k dt$$

Let's define  $S_0 = A_0 + B_0$

Then the above equation becomes

$$\frac{d[B]}{[B](S_0 - [B])} = -k dt$$

$$\rightarrow \frac{d[B]}{[B]} + \frac{d[B]}{(S_0 - [B])} = -kS_0 dt$$

$$\rightarrow \int \frac{d[B]}{[B]} + \int \frac{d[B]}{(S_0 - [B])} = -\int kS_0 dt$$

$$\rightarrow \ln[B] - \ln(S_0 - [B]) + C = -kS_0 t + C$$

$$\rightarrow [B]/(S_0 - [B]) = C' e^{-kS_0 t}$$

Apply the initial condition to the above equation

$$t=0 \rightarrow [B] = B_0$$

$$\rightarrow C' = \frac{B_0}{S_0 - B_0} = \frac{B_0}{A_0}$$

$$\text{And then } [B]/(S_0 - [B]) = \frac{A_0}{B_0} e^{-kS_0 t}$$

Substitute  $S_0 = A_0 + B_0$  into the above equation

$$[B]/(A_0 + B_0 - [B]) = \frac{A_0}{B_0} e^{-k(A_0 + B_0)t}$$

By isolating [B] into one side, you should be able to get [B]

$$[B] = \frac{A_0 + B_0}{1 + \frac{A_0}{B_0} e^{k(A_0 + B_0)t}}$$

$$[A] = A_0 + B_0 - [B]$$

$$\rightarrow [A] = \frac{A_0 + B_0}{1 + \frac{B_0}{A_0} e^{-k(A_0 + B_0)t}}$$